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"The question isn't who is going to let me; it's who is going to stop me."-Ayn Rand

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AIM: How do we find the sum and product of the roots of a quadratic equation?

Warm Up:

1) What value(s) of "c" would give us an equation with imaginary roots?

$$\overset{a}{1}x^2 + \overset{b}{2}x + \overset{c}{c} = 0$$

Discriminant
b^2-4ac

$b^2-4ac < 0$

$2^2-4(1)(c) < 0$

$4-4c < 0$

$\quad +4c \quad +4c$

$\frac{4}{4} < \frac{4c}{4}$

$1 < c$

$c > 1$

As long as the value of "c" is greater than 1 in this example the roots will be imaginary.

Consider the quadratic:

$$ax^2+bx+c=0$$

Represent the roots as:

$$r_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

and $$r_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of the Roots

Add the roots:

$$r_1+r_2=\frac{-b+\sqrt{b^2-4ac}}{2a}+\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b\cancel{+\sqrt{b^2-4ac}}-b\cancel{-\sqrt{b^2-4ac}}}{2a}$$

$$=\frac{-\cancel{2}b}{\cancel{2}a}=\frac{-b}{a}$$

= Sum of roots

Product of the Roots

Multiply the roots:

$$r_1\cdot r_2=\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\cdot\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$=\frac{\cancel{b^2}\cancel{+b\sqrt{b^2-4ac}}\cancel{-b\sqrt{b^2-4ac}}-(\cancel{b^2}+4ac)}{4a^2}$$

$$=\frac{b^2-b^2+4ac}{4a^2}=\frac{\cancel{4}ac}{\cancel{4}a^{\cancel{2}}}=\frac{c}{a}$$

= Product of roots

☆ [redacted]: If the quadratic equation has rational coefficients then the roots will occur in conjugate pairs. Ex: $(1+i)$ and $(1-i)$

$(x+2)(x-2)$ $(1+\sqrt{2})$ and $(1-\sqrt{2})$

Find the sum and product of the roots of the following equations:

2. $x^2+4x+5=0$

$a=1$ $\quad$ $\text{sum} = \frac{-b}{a} = \frac{-4}{1} = \boxed{-4}$

$b=4$

$c=5$ $\quad$ $\text{Product} = \frac{c}{a} = \frac{5}{1} = \boxed{5}$

3. $2x^2-6x+10=0$

$a=2$ $\quad$ $\text{sum} = \frac{-b}{a} = \frac{-(-6)}{2} = \frac{6}{2} = \boxed{3}$

$b=-6$

$c=10$ $\quad$ $\text{product} = \frac{c}{a} = \frac{10}{2} = \boxed{5}$

4. $4x^2=2x+9$

$$\begin{array}{r} 4x^2 = 2x+9 \\ -2x-9 \quad -2x-9 \\ \hline 4x^2-2x-9=0 \end{array}$$

$a=4$ $\quad$ $\text{Sum} = \frac{-b}{a} = \frac{-(-2)}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

$b=-2$

$c=-9$ $\quad$ $\text{Prod} = \frac{c}{a} = \boxed{\frac{-9}{4}}$

Alt:

$$\begin{array}{r} 4x^2 = 2x+9 \\ -4x^2 \quad -4x^2 \\ \hline 0 = -4x^2+2x+9 \end{array}$$

$a=-4$ $\quad$ $\text{sum} = \frac{-b}{a} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$

$b=2$

$c=9$ $\quad$ $\text{prod} = \frac{c}{a} = \boxed{\frac{9}{-4}}$

5. If the sum of the roots is 12 and one of the roots is 5, what is the other root?

$r_1 + r_2 = \text{sum}$

$5 + r_2 = 12$

$-5 \qquad\quad -5$

$\boxed{r_2 = 7}$

6. Given the equation $x^2 + 15x + c = 0$
If r_1 is -5, what is the other root?

$a = 1$

$b = 15$

$\text{sum} = \frac{-15}{1} = -15$

$r_1 + r_2 = \text{sum}$

$-5 + r_2 = -15$

$+5 \qquad\quad +5$

$\boxed{r_2 = -10}$

7. $x^2 + bx + 3 \overset{=0}{;} \; r_1 = 1$

What is the other root?
What is the value of b?

Use either root for x

$a = 1$

$b = ?$

$c = 3$

$\text{Product} = \frac{c}{a} = \frac{3}{1}$

$r_1 \cdot r_2 = \text{product}$

$1 \cdot r_2 = 3$

$\boxed{r_2 = 3}$

$3^2 + b(3) + 3 = 0$

$9 + 3b + 3 = 0$

$12 + 3b = 0$

$-12 \qquad -12$

$\frac{3b}{3} = \frac{-12}{3}$

$\boxed{b = -4}$

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	Sum $= \frac{-b}{a}$	Product $= \frac{c}{a}$
3)	-1	1
5) $a=2$, $b=-3$, $c=-2$	$\frac{-(-3)}{2} = \frac{3}{2}$	$\frac{-2}{2} = -1$
7) $a=3$, $b=-6$, $c=4$	$\frac{-(-6)}{3} = \frac{6}{3} = 2$	$\frac{4}{3}$
9) $a=1$, $b=-8$, $c=-12$	$\frac{-(-8)}{1} = \frac{8}{1} = 8$	$\frac{-12}{1} = -12$
11) $a=2$, $b=-5$, $c=-8$	$\frac{-(-5)}{2} = \frac{5}{2}$	$\frac{-8}{2} = -4$
13) $a=1$, $b=0$, $c=1$	$\frac{-(0)}{1} = 0$	$\frac{1}{1} = 1$
15) $a=8$, $b=6$, $c=-9$	$\frac{-(6)}{8} = \frac{-3}{4}$	$\frac{-9}{8}$
17) $a=3$, $b=3$, $c=5$	$\frac{-3}{3} = -1$	$\frac{5}{3}$

$$\frac{3x^2+3x+5}{3} = \frac{0}{1}$$

$$(3)\ \frac{3x^2+3x+5}{3} = 0\ (3)$$

$$3x^2+3x+5=0$$

1) What is the sum and product of the roots?

$$3x^2+8=5x$$

2) What is the nature of the roots? (Describe them)

a) real, rational, equal

b) real, irrational, unequal

c) real, rational, unequal

d) imaginary